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F. Ye, M. Matsuda, S. Katano, H. Yoshizawa, D. P. Belanger, E. T. Seppala, J. A. Fernandez-Baca, M. J. Alava

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F. Ye [a] M. Matsuda [b] S. Katano [b] H. Yoshizawa [c] D. P. Belanger [a,*] E. T. Seppälä [e] J. A. Fernandez-Baca [d] M. J. Alava [f]

[a] *Department of Physics, University of California, Santa Cruz, CA 95064 USA*
[b] *Advanced Science Research Center, Japan Atomic Energy Research Institute, Tokai, Ibaraki 319-1195 Japan*
[c] *Neutron Science Lab., Institute for Solid State Physics, Univ. of Tokyo, Shirakata 106-1, Tokai, Ibaraki 319-1106 Japan*
[d] *Condensed Matter Sciences Div., Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831 USA*
[e] *Lawrence Livermore National Laboratory, L-415, Livermore, CA 94551 USA*
[f] *Dipartimento di Fisica, Università "La Sapienza", P. le A. Moro 2, 00185 Roma, Italy*

Abstract

Neutron scattering and simulation line shape data show evidence for fractal structure from spanning clusters in the $d = 2$ and $d = 3$ random-field Ising models as realized in dilute antiferromagnets.

Key words: Random-field Ising model, neutron scattering, percolation
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Rigorous proofs of the two (2D) and three dimensional (3D) random-field Ising model (RFIM) indicate that there is a phase transition in 3D, but not in 2D [1]. However, recent simulations show that in both dimensions there is a percolation type of order: in 2D at low random-field values and in 3D just above the phase transition [2]. The importance of the percolation phenomenon is not only of pure theoretical interest, but it may also have implications in the interpretation of experimental results such as line shapes from the neutron scattering experiments due to the fractal character of the spanning cluster. This can be tested in the simulations as well, where the Fourier transform of spin-spin correlation functions, representing the structure factor $S(q)$, shows scaling with the percolation fractal dimension $D_f = 91/48$ (see Fig. 1). In the figure $S(q)$ is calculated from ground states of 2D RFIM with diluted antiferromagnet type of disorder, where the magnetic dilution is $1 - x = 0.30$, and square-lattices. The ground states are found using an exact optimization algorithm.

Experimental scattering intensity line shapes are given by $S(q)$ folded with instrumental resolution. One expects $S(q)$ for $T > T_c$ to be governed by a scaling function which can be approximated by [3]

$$f = A\kappa^{\eta-2}(1 + \phi^2 q^2/\kappa^2)^{\eta/2}/(1 + \psi^2 q^2/\kappa^2) \ , \quad (1)$$

where $\psi = 1 + \frac{1}{2}\eta\phi^2$ and κ is the inverse fluctuation

* Corresponding author, Tel: +1 831 459 2871; Fax: +1 831 459 3043
Email address: dave@dave.ucsc.edu (D. P. Belanger).

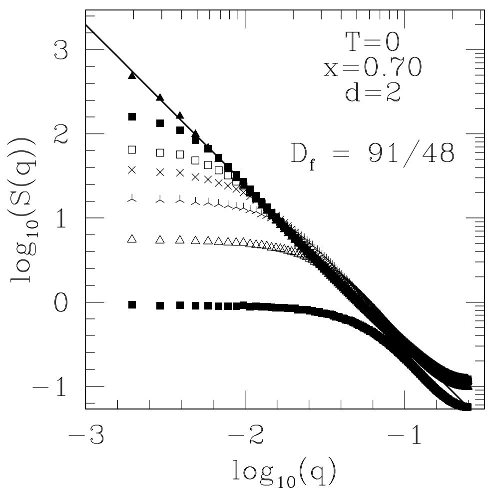

Fig. 1. Simulation results for $\log_{10}(S(q))$ vs. $\log_{10}|q|$ for $d = 2$. The solid line is $q^{-91/48}$. Data sets, highest to lowest, are for $H/J = 0.50$, 0.75, 1.00, 1.25, 1.75, 2.50, and 3.25, respectively.

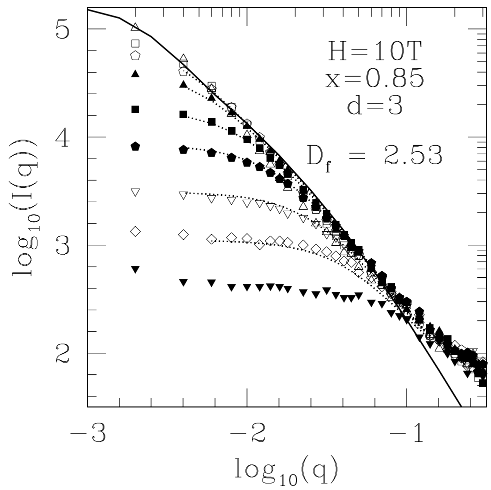

Fig. 2. Scattering results for $\log_{10}(I(q))$ vs. $\log_{10}|q|$ for $d = 3$ in $Fe_{0.85}Zn_{0.15}F_2$ at $t = 3.9 \times 10^{-4}$, 3.5×10^{-3}, 5.1×10^{-3}, 7.5×10^{-3}, 1.2×10^{-2}, 1.8×10^{-2}, 3.1×10^{-2}, 5.0×10^{-2}, and 1.0×10^{-1} (lowest set). The solid curve is $q^{-2.53}$, folded with instrumental resolution. The dotted curves are fits to Eq. (1).

correlation length. In the mean-field approximation (MF), $\eta = 0$, yielding a simple Lorentzian. For $H = 0$, theory and experiment show that $S(q) \approx f$ with η very small. However, the RFIM result in MF is $S(q) = f + R(H)f^2$ with $\eta = 0$.

Experimental RFIM data for $H = 10$ T in $\mathrm{Fe_{0.85}Zn_{0.15}F_2}$ were obtained at 13.7 meV using the JAERI TAS-2 neutron spectrometer with pyrolytic graphite (PG) monochromator and analyzer (002) reflections and a PG filter. Half-width-at-half-maximum resolutions were 0.0019, 0.0084 and 0.052 reciprocal lattice units in the transverse, longitudinal and vertical directions, respectively.

Using techniques developed previously [3], fits to $S(q) = f + R(H)f^2$ for $t < 4.5 \times 10^{-2}$ and $4 \times 10^{-3} < |q| < 3 \times 10^{-1}$ yield $\eta = 0.58 \pm 0.05$ and $\nu = 1.18 \pm 0.05$, but do not seem to yield convergence to a universal scaling function f as H is increased, as one might have expected. Motivated by the simulations described above, fits to $S(q) = f$ for $t < 6 \times 10^{-2}$ and $2 \times 10^{-3} < |q| < 1.5 \times 10^{-1}$ were made for $T > T_c = 63.6K$, and $H = 10T$, as shown in Fig. 2, giving exponents $\eta = -0.5 \pm 0.05$ and $\nu = 1.20 \pm 0.05$. As $t = (T - T_c)/T_c \to 0$, the line shapes follow the fractal line shape $S(q) \to |q|^{-D_f}$, shown folded with the instrumental resolution by the solid curve in Fig. 2, where $D_f = 2 - \eta = 2.53$ for $d = 3$. This suggests that $S(q)$ is strongly influenced by the fractal character of the spanning clusters, as simulations suggest. For $H > 0$ and $T < T_c$, and for $H = 0$, no RFIM spanning clusters are expected to exist and experimental line shapes in these cases are not fractal powers of q.

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